Global sensitivity analysis for the geostatistical characterization of a regional-scale sedimentary aquifer

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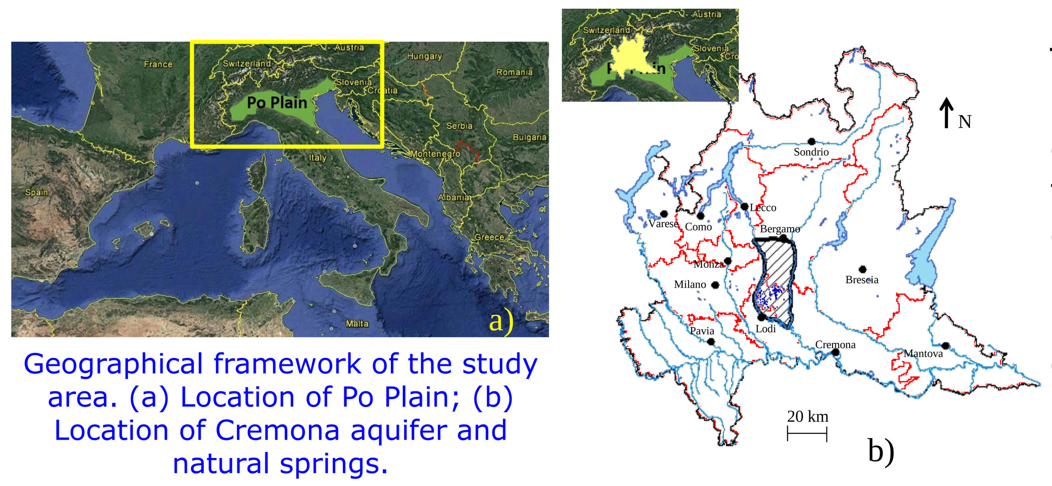
WE-NEED
WatEr NEEDs, Availability, Quality and Sustainability

1. STUDY AREA AND AVAILABLE DATA SET

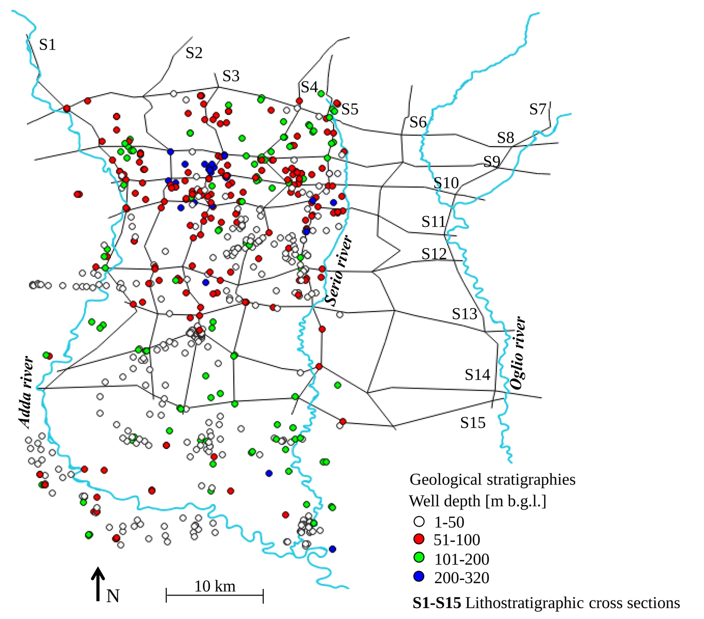


Geographical framework of the study area. (a) Location of Po Plain; (b) Location of Cremona aquifer and natural springs.

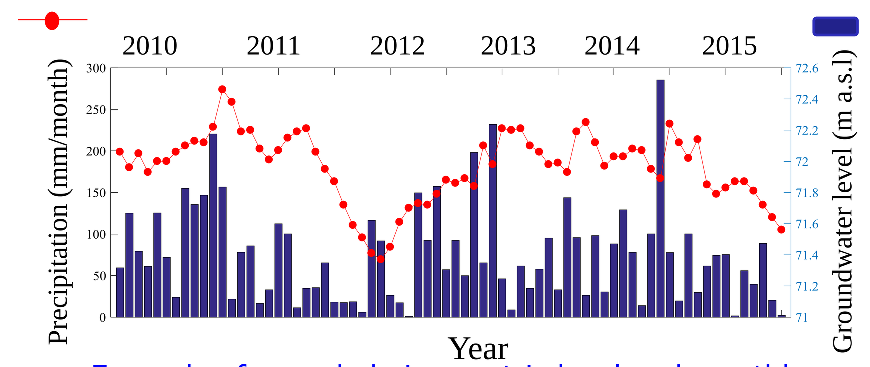
The study aquifer lies within the provinces of Bergamo and Cremona (Italy) and covers a planar extent of approximately 785 km^2 . A key feature of the study area is the occurrence of natural high-quality water springs.

Data available: geological (e.g. lithostratigraphic-cross sections and geological stratigraphies) and hydrological data (e.g. precipitation, temperature, piezometric level, hydrometric level, water concession at pumping wells).

GEOLOGICAL DATA



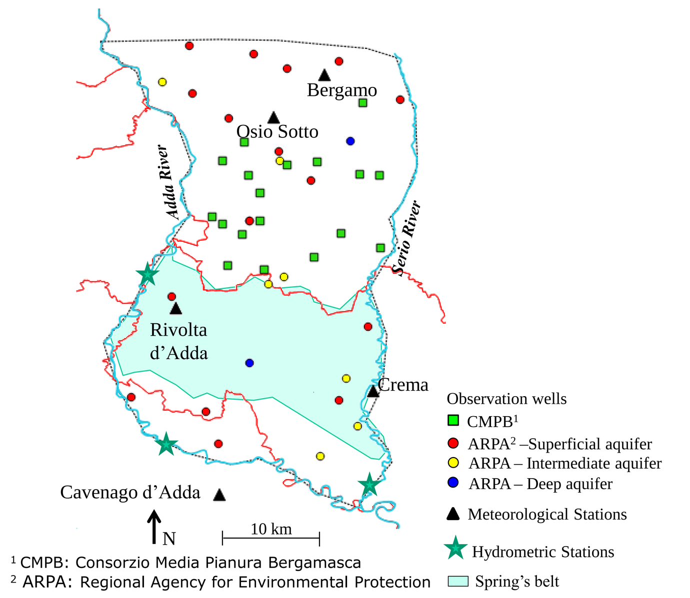
Location of lithostratigraphic cross sections and geological stratigraphies within the study area



Example of recorded piezometric level and monthly precipitation at the station Rivolta d'Adda

Analysis of sedimentological information allows identifying 5 main geo-materials (facies/phases)

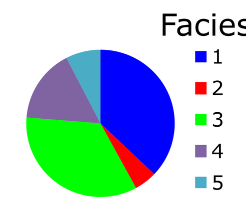
HYDROLOGICAL DATA



[1] CMPB: Consorzio Media Pianura Bergamasca
[2] ARPA: Regional Agency for Environmental Protection

Location of hydrological measurement stations within the study area

CLASS	GEOLOGICAL MATERIAL
1	Clay and silt
2	Fine and silty sand
3	Gravel, sand and gravel
4	Compact conglomerate
5	Fractured conglomerate

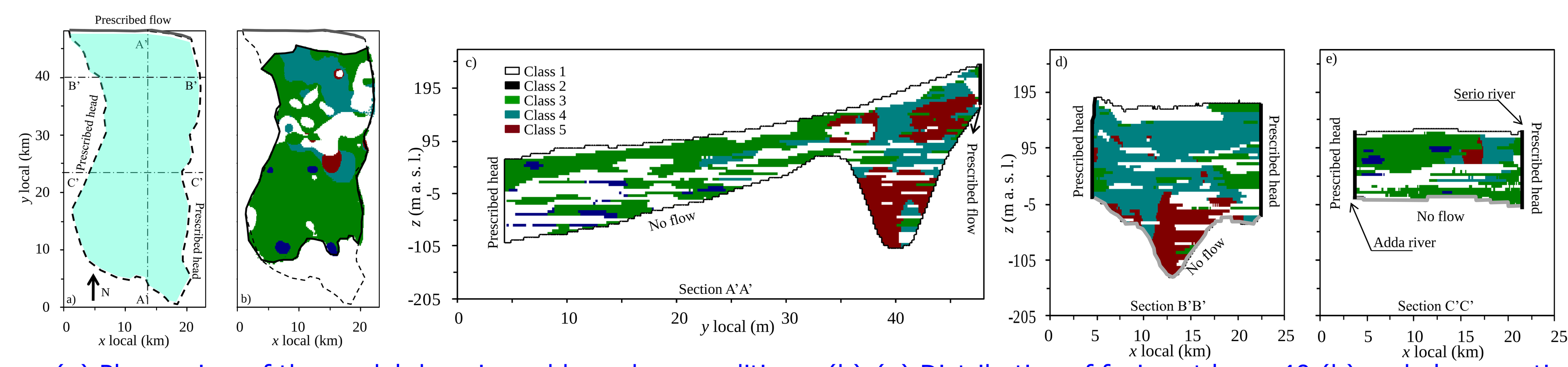


Geological composition and volumetric percentage of the five geological facies

These five classes form the basis for the definition of indexed variables by which we describe the distribution of geo-materials in the aquifer.

2. GEOLOGICAL RECONSTRUCTIONS – GROUNDWATER FLOW MODEL

Domain size: 23 km (E-W direction) × 48 km (N-S direction) × 475 m (depth). The system is discretized into 95 layers, 230 columns and 240 rows. Each cell has dimension 100 m × 200 m × 5 m. The numerical code MODFLOW-2005 is used to simulate groundwater steady state flow. We parameterize the conductivity field following two different conceptual schemes: 'Composite Medium' and 'Overlapping Continuum'.



(a) Planar view of the model domain and boundary conditions. (b)-(e) Distribution of facies at layer 40 (b) and along sections AA' (c) BB' (d) and CC' (e)

A. COMPOSITE MEDIUM

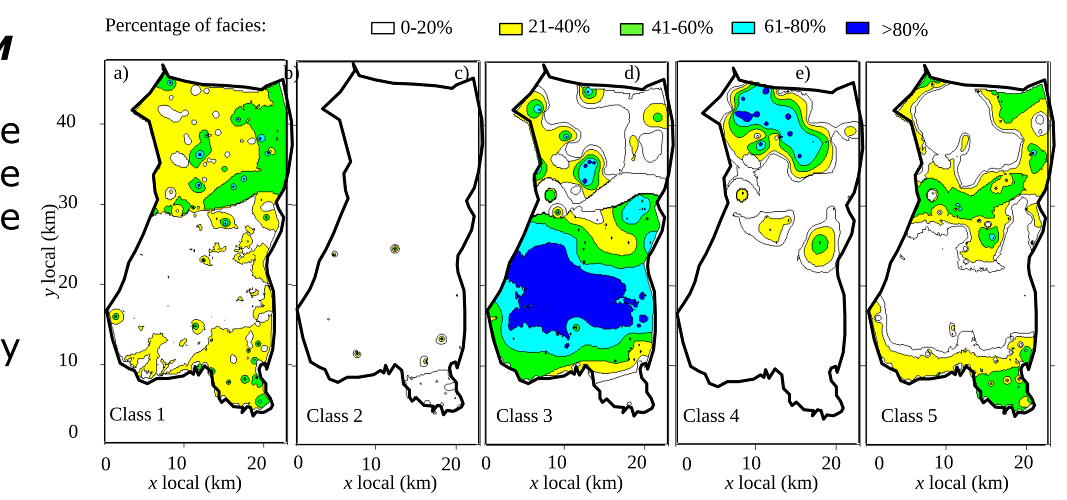
The system is composed by distinct (five) lithological units. Hydraulic properties (such as conductivity) in each unit are assumed to be uniform. The lithotypes reconstruction is based on indicator kriging (e.g., Guadagnini et al., 2004).

B. OVERLAPPING CONTINUUM

The system is modelled as many composite media coexisting in space. The rationale of this model is based on considering each cell of the domain formed by multiple lithofacies. In this framework the volumetric fraction of a lithofacies can be interpreted as its probability of occurrence.

The conductivity of each cell of the model is a weighted mean of the conductivity values of the 5 classes.

ARITHMETIC MEAN: $K_{cell} = \sum_{i=1}^{5} k_i\, p_i$ GEOMETRIC MEAN: $K_{cell} = \prod_{i=1}^{5} k_i\, p_i$



Percentage of 5 facies within layer 40

3. SENSITIVITY ANALYSIS - METHODOLOGY

MORRIS SCREENING METHOD

Considering the vector β = (β_1,…,β_N), whose elements are the *N* parameters which are investigated as uncertain, the elementary effect (*EE*) of the *i*-th parameter is defined as

$$EE_i = \frac{f(\beta_1,\dots,\beta_i+\Delta,\dots,\beta_N) - f(\boldsymbol{\beta})}{\Delta}$$

where Δ is the parameter increment and *f*(**β**) is the model response for the parameter values **β**. The sensitivity measure μ^* can be calculated as (Campolongo et al., 2007)

$$\mu_i^* = \frac{1}{r}\sum_{j=1}^{r}\left|EE_i(j)\right|$$

where *r* is the number of trajectories of sample points in the parameter space, $EE_i(j)$ is the elementary effect of the *i*-th parameter and the *j*-th trajectory.

SOBOL INDEXES

The total variance V_f of the model response *f*(**β**) can be written as

$$V_f = \sum_{i=1}^{N} V_{f,i} + \sum_{1\le i<j\le N} V_{f,ij} + \dots + V_{f,1,\dots,N}$$

here, $V_{f,i}$ is the contribution to the variance of the model output due to the effect of the uncertain input parameter β_i when considered individually, and $V_{f,i_1,\dots,i_s}$ is due to interaction of the uncertain model parameters belonging to the subset $\{\beta_{i_1},\dots,\beta_{i_s}\}$. Sobol indices are defined as (Sobol, 1993)

$$S(f,\beta_{i_1},\dots,\beta_{i_s}) = \frac{V_{f,i_1,\dots,i_s}}{V_f}$$

and express the contribution of a subset of model parameters $\{\beta_{i_1},\dots,\beta_{i_s}\}$ to the total model variance.

UNCERTAIN PARAMETERS

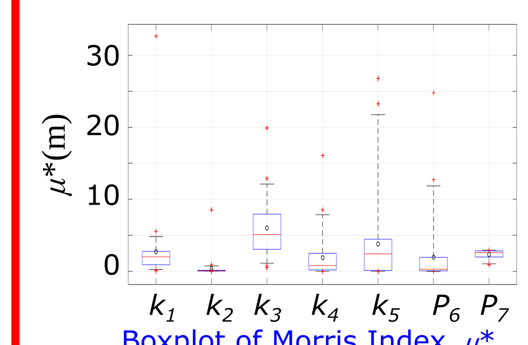
Parameter	Description	Lower bound	Upper bound	Unit
k_1	Conductivity of geomaterial 1	10^{-8}	10^{-5}	m/s
k_2	Conductivity of geomaterial 2	10^{-7}	10^{-4}	m/s
k_3	Conductivity of geomaterial 3	10^{-4}	10^{-2}	m/s
k_4	Conductivity of geomaterial 4	10^{-6}	10^{-3}	m/s
k_5	Conductivity of geomaterial 5	10^{-3}	10^{-1}	m/s
P_6	Prescribed flow (BC)	4.83	19.30	m^3/s
P_7	Prescribed head (BC)	0.0	3.0	m

Uncertain parameters considered in the sensitivity analysis with lower and upper bounds

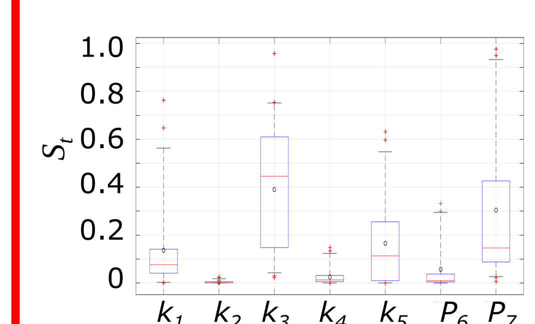
4. RESULTS

The sensitivity analysis have been performed to study the effect of the uncertain parameters on the hydraulic head monitored at 40 observation wells.

COMPOSITE MEDIUM



Boxplot of Morris Index μ^*

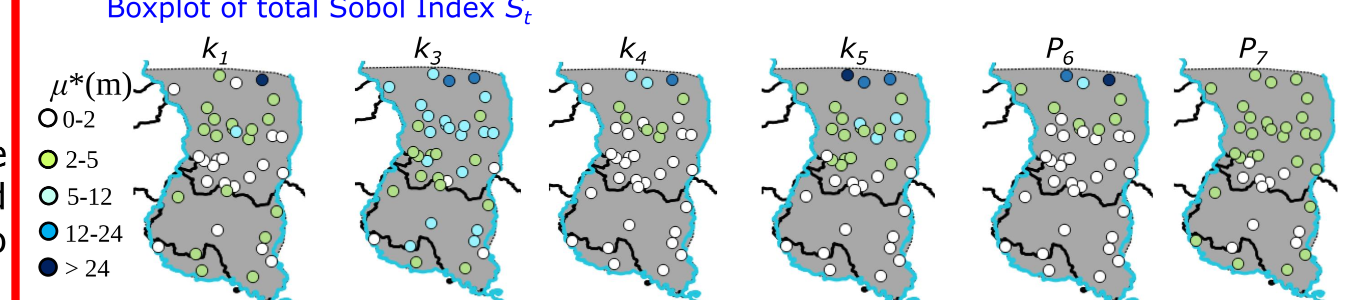


Boxplot of total Sobol Index S_t

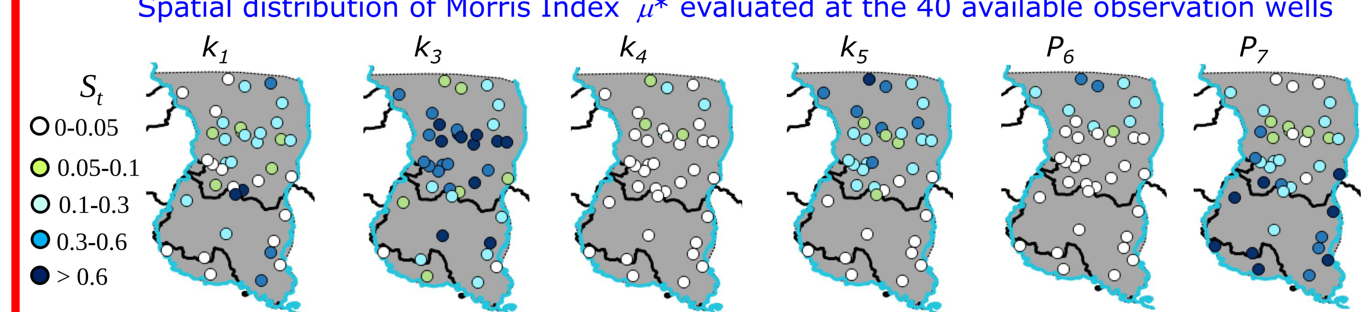
Morris approach is characterized by a low computational cost (240 model runs) with respect to the Sobol indexes evaluation, computed via a model reduction technique, i.e. generalized polynomial chaos expansion (2437 model runs).

Outcomes are not significantly affected by k_2 and k_4. This result is consistent with the observation that two geomaterials constitute only 5% and 16% of the system.

Boundary condition P_6 is important only close to the northern boundary.

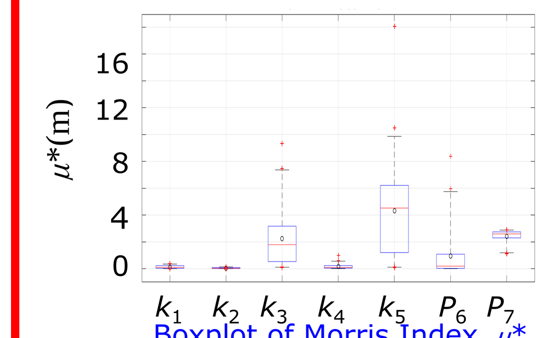


Spatial distribution of Morris Index μ^* evaluated at the 40 available observation wells

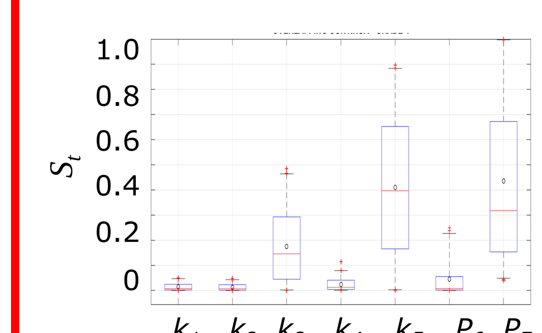


Spatial distribution of total Sobol Index S_t evaluated at the 40 available observation wells

OVERLAPPING CONTINUUM - ARITHMETIC MEAN



Boxplot of Morris Index μ^*



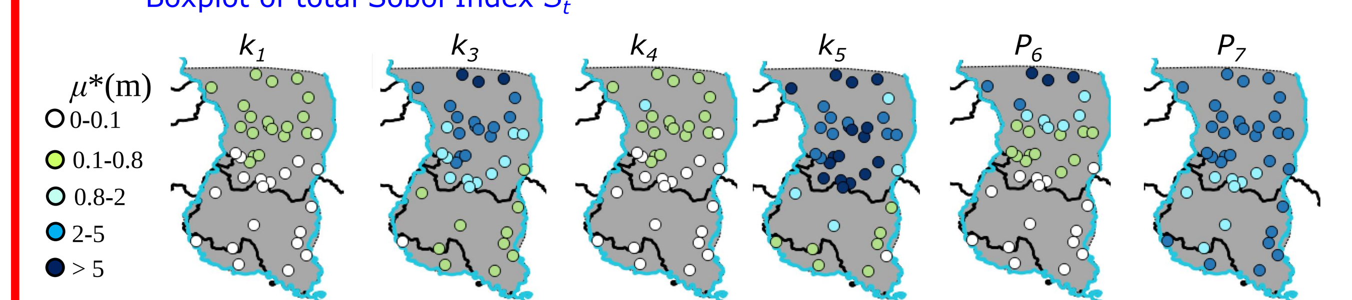
Boxplot of total Sobol Index S_t

Sobol and Morris sensitivity indexes identify consistent ranking of model parameters.

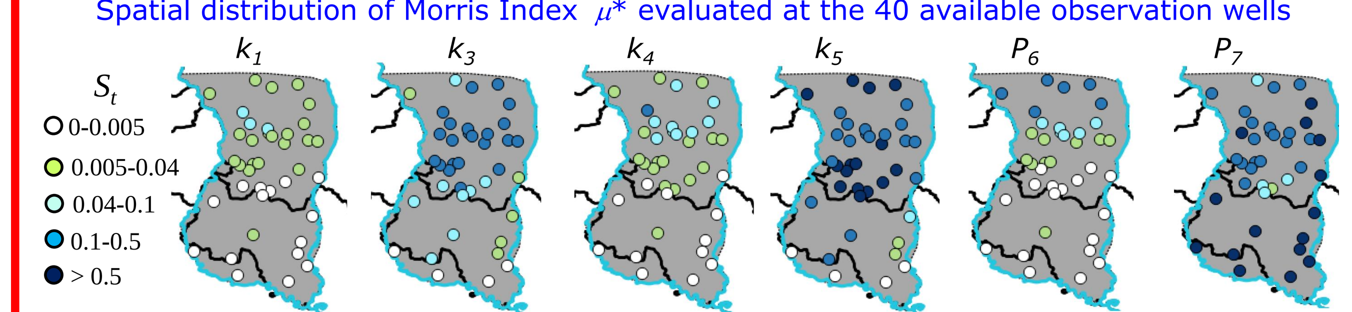
Hydraulic heads are not affected by variation of k_1, k_2 and k_4. This result is related to the fact that arithmetic mean tends to reduce the effect of the low conductivity values (associated with clay, silt and compact conglomerates).

Boundary condition P_7, k_3 and k_5, strongly affect the outcome of the model.

Boundary condition P_6 is important only close to the northern boundary.

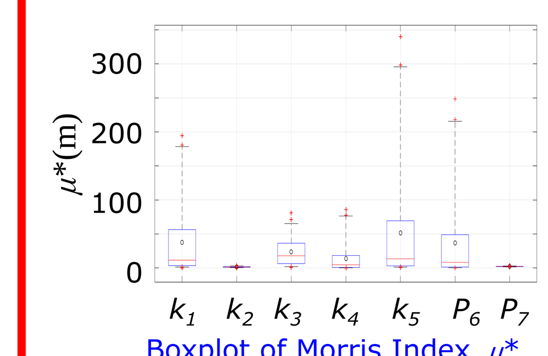


Spatial distribution of Morris Index μ^* evaluated at the 40 available observation wells

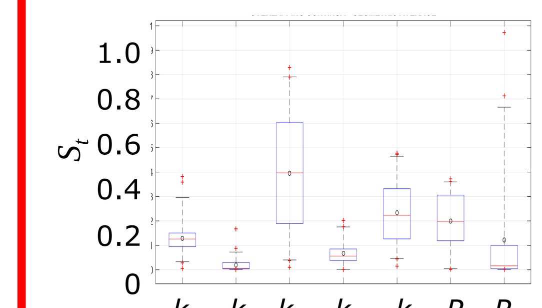


Spatial distribution of total Sobol Index S_t evaluated at the 40 available observation wells

OVERLAPPING CONTINUUM - GEOMETRIC MEAN



Boxplot of Morris Index μ^*

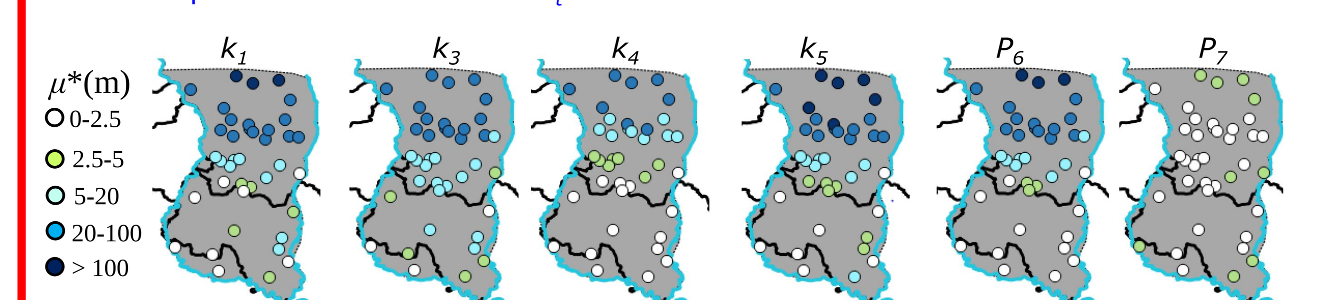


Boxplot of total Sobol Index S_t

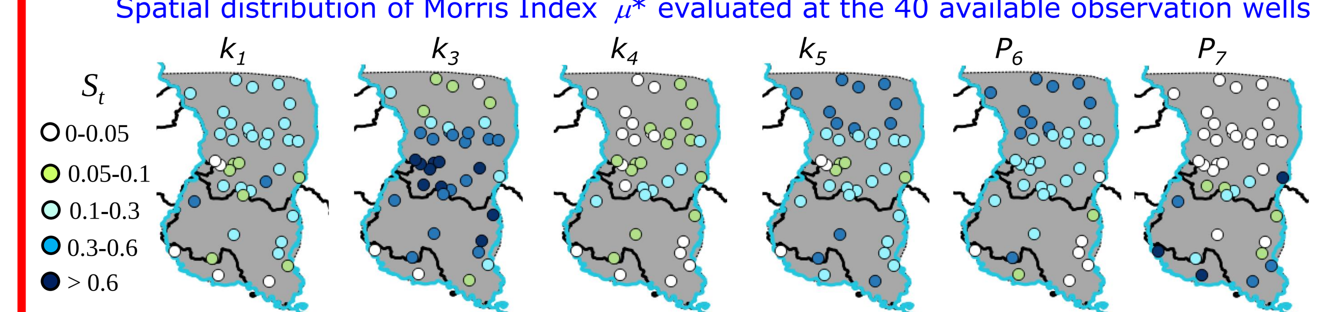
Hydraulic heads display wider variation with respect to the other two considered models.

Conductivity values that mostly affect the model outcomes are k_1, k_3 and k_5 (clay, gravel and compact conglomerate). The same results was observed for the *Composite Medium* model.

Boundary condition P_6 becomes more important than it was for the other two models while boundary condition P_7 does not significantly affect model outcomes.



Spatial distribution of Morris Index μ^* evaluated at the 40 available observation wells



Spatial distribution of total Sobol Index S_t evaluated at the 40 available observation wells

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L. Guadagnini, A. Guadagnini, D.M. Tartakowsky (2004) Probabilistic reconstruction of geological facies. *J. Hydrol.* 294, 57-67.
I.M. Sobol (1993) Sensitivity indices for nonlinear mathematical models. *Math. Mod. Comput. Exp.* , 39-56.

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